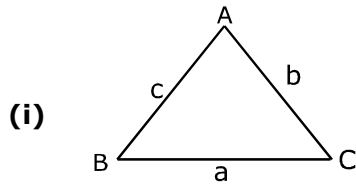


EXERCISE – V**HINTS & SOLUTIONS****Sol.1**

$$\text{Area} = \frac{1}{2}(\vec{a} \times \vec{b}) = \frac{1}{2}(\vec{b} \times \vec{c}) = \frac{1}{2}(\vec{c} \times \vec{a})$$

(ii) Normal vector of $P_1 = \vec{a} \times \vec{b}$ Normal vector of $P_2 = \vec{c} \times \vec{d}$ given that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0$ that means \vec{P}_1 is parallel to \vec{P}_2 so angle = 0° **(iii)** $|\vec{a}| = |\vec{b}| = |\vec{c}|$

$$[\vec{a} \vec{b} \vec{c}] = 0 \quad (\text{given})$$

$$[2\vec{a} - \vec{b} \quad 2\vec{b} - \vec{c} \quad 2\vec{c} - \vec{a}]$$

$$(2\vec{a} - \vec{b}) \cdot [(2\vec{b} - \vec{c}) \times (2\vec{c} - \vec{a})]$$

$$= (2\vec{a} - \vec{b}) \cdot [4(\vec{b} \times \vec{c}) - (\vec{b} \times \vec{a}) + (\vec{c} \times \vec{a})]$$

$$= 8[\vec{a} \vec{b} \vec{c}] - 0 + 0 - 0 - 0 - [\vec{b} \vec{c} \vec{a}] = 0$$

Sol.2 (i) $\vec{a} = (1, 1, -1); \vec{b} = (-1, 2, 2);$

$$\vec{c} = (-1, 2, -1)$$

$$\vec{a} + \vec{b} = (0, 3, 1); \quad \vec{b} - \vec{c} = (0, 0, 3)$$

$$\text{Normal vector} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{vmatrix}$$

$$= 9\hat{i} - 0\hat{j} + 0\hat{k}$$

$$\text{unit normal vector} = \pm \hat{i}$$

(ii) $\vec{a} \cdot \vec{b} = 0$ so, \vec{a} , \vec{b} and $\vec{a} \times \vec{b}$ are perpendicular to each other.

$$\vec{V} = p\vec{a} + q\vec{b} + r(\vec{a} \times \vec{b})$$

$$\vec{V} \cdot \vec{a} = 0 \Rightarrow p(\vec{a} \cdot \vec{a}) = 0 \Rightarrow p = 0$$

$$\vec{V} \cdot \vec{b} = 1 \Rightarrow 0 + q|\vec{b}|^2 + 0 = 1$$

$$\Rightarrow q = \frac{1}{|\vec{b}|^2}$$

$$\vec{V} \cdot (\vec{a} \times \vec{b}) = p \cdot 0 + q \cdot 0 + r|\vec{a} \times \vec{b}|^2$$

$$\Rightarrow \frac{[\vec{a} \vec{b}]}{|\vec{a} \times \vec{b}|^2} = \frac{1}{|\vec{a} \times \vec{b}|^2}$$

$$\vec{V} = \frac{\vec{b}}{|\vec{b}|^2} + \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|^2}$$

(iii) $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \vec{c}$

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{1}{2}(\vec{b} + \vec{c})$$

$$\vec{a} \cdot \vec{c} = \frac{1}{2} \quad \vec{a} \cdot \vec{b} = -\frac{1}{2}$$

$$|\vec{a}| |\vec{b}| \cos \theta = \frac{-1}{2}$$

$$\Rightarrow \cos \theta = \frac{-1}{2} \Rightarrow \theta = \frac{2\pi}{3} \quad \text{Ans.}$$

Sol.3 (a) $\vec{d}_1 = (2, 3, -6); \quad \vec{d}_2 = (3, -4, -1)$

$$\text{Area} = \frac{1}{2} |\vec{d}_1 \times \vec{d}_2| = \frac{1}{2} \sqrt{1274}$$

If sides are \vec{a} and \vec{b}

$$\vec{d}_1 = \vec{a} + \vec{b}; \quad \vec{d}_2 = \vec{a} - \vec{b}$$

$$\vec{a} = \frac{\vec{d}_1 + \vec{d}_2}{2}; \quad \vec{b} = \frac{\vec{d}_1 - \vec{d}_2}{2}$$

$$= \frac{1}{2} (5, -1, -7) \quad = \frac{1}{2} (-1, 7, -5) \text{ Ans.}$$

(b) $(-4, 5, 0)a + (3, -3, 1)b + (1, 1, 3)c = \lambda(a, b, c)$

$$(-4a + 3b + c - \lambda a)\hat{i} + (5a - 3b + c - \lambda b)\hat{j} + (b + 3c - \lambda c)\hat{k} = 0$$

$$\ell\hat{i} + m\hat{j} + n\hat{k} = 0$$

$$\ell = m = n = 0$$

$$-4a + 3b + c - \lambda a = 0$$

$$5a - 3b + c - \lambda b = 0$$

$$b + 3c - \lambda c = 0$$

$$\text{again } (-4 - \lambda)a + 3b + c = 0$$

$$5a + (-3 - \lambda)b + c = 0$$

$$0a + b + (3 - \lambda)c = 0$$

$$\begin{vmatrix} -4-\lambda & 3 & 1 \\ 5 & -3\lambda & 1 \\ 0 & b & 3-\lambda \end{vmatrix} = 0 \Rightarrow \lambda = 0, -2 \pm \sqrt{29}$$

$$\text{Sol.4 (a)} \vec{r} = \lambda(\vec{a} \times \vec{b})$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 5 \\ 2 & 3 & -1 \end{vmatrix}$$

$$\vec{r} = \lambda(-13, -11, 7)$$

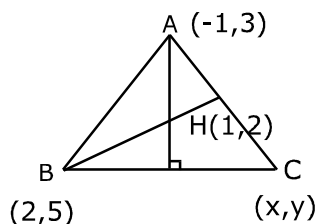
$$= \hat{i}(2 - 15) - \hat{j}(-1 - 10) + \hat{k}(3 + 4)$$

$$= -13\hat{i} + 11\hat{j} + 7\hat{k}$$

$$\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$$

$$\lambda = 1 \text{ so } \vec{r} = (-13, -11, 7)$$

(b) H is orthocentre



$$\vec{AH} \cdot \vec{BC} = 0$$

$$(2, -1) \cdot (x - 2, y - 5) = 0$$

$$2x - y + 1 = 0 \quad \dots (1)$$

$$\vec{BH} \cdot \vec{AC} = 0$$

$$(-1, -3) \cdot (x + 1, y - 3) = 0$$

$$-x - 1 - 3y + 9 = 0$$

$$x + 3y = 8 \quad \dots (2)$$

$$\text{from (1) and (2) } x = \frac{5}{7}; y = \frac{17}{7}$$

$$C\left(\frac{5}{7}, \frac{17}{7}\right)$$

$$\text{Sol.5 (a)} (\vec{a} + \vec{b} + \vec{c})^2 = \sum \vec{a}^2 + 2\sum \vec{a} \cdot \vec{b} \geq 0$$

$$2\sum \vec{a} \cdot \vec{b} \geq -3(\because |\vec{a}| = |\vec{b}| = |\vec{c}| = 1)$$

$$\sum |\vec{a} - \vec{b}|^2 = 2\sum \vec{a}^2 - 2\sum \vec{a} \cdot \vec{b}$$

$$\leq 2(3) + 3 \leq 9$$

$$\text{(b)} [\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

Applying $C_3 \rightarrow C_1 + C_3$

$$= \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ y & x & 1+x \end{vmatrix} = 1$$

Sol.6 $\vec{A}(t)$ is parallel to $\vec{B}(t)$ only if

$$\frac{f_1(t)}{g_1(t)} = \frac{f_2(t)}{g_2(t)} \text{ for some } t \in [0, 1]$$

$$\text{or } f_1(t) \cdot g_2(t) = f_2(t) \cdot g_1(t)$$

$$\text{let } h(t) = f_1(t) \cdot g_2(t) - f_2(t) \cdot g_1(t)$$

$$h(0) = 2 \times 2 - 3 \times 3 = -5 < 0$$

$$h(1) = f_1(1) \cdot g_2(1) - f_2(1) \cdot g_1(1)$$

$$= 6 \times 6 - 2 \times 2 = 32 > 0$$

Since, h is a continuous function and $h(0) \cdot h(1) < 0$

\Rightarrow there is some $t \in [0, 1]$ for which $-h(t) = 0$

$$\text{Sol.7 (a)} (\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 4\vec{b}) = 0$$

$$5|\vec{a}|^2 - 4\vec{a} \cdot \vec{b} + 10\vec{b} \cdot \vec{a} - 8|\vec{b}|^2 = 0$$

$$6\vec{a} \cdot \vec{b} = 3 \Rightarrow \vec{a} \cdot \vec{b} = \frac{1}{2}$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

$$\text{(b)} \vec{v} = (2, 1, -1) \quad \vec{w} = (1, 0, 3)$$

$$[\vec{u} \vec{v} \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$$

$$= \vec{u} \cdot (3, -7, -1) = \vec{u} \cdot |3, -7, -1| \cos \theta$$

Maximum value of box product

$$= |3, -7, -1| = \sqrt{9 + 49 + 1} = \sqrt{59}$$

$$\text{Sol.8} \quad [\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = a + a^3 - a$$

$$f(a) = a^3 - a + 1$$

$$f'(a) = 3a^2 - 1 = 0 \Rightarrow a = \pm \frac{1}{\sqrt{3}}$$

minimum value of cubic in real line can't be determine

$$\text{Sol.9} \quad \vec{x} = \frac{\vec{u} + \vec{v}}{|\vec{u} + \vec{v}|} = \frac{1}{2} \sec \frac{\alpha}{2} (\vec{u} + \vec{v})$$

$$\vec{y} = \frac{\vec{v} + \vec{w}}{|\vec{v} + \vec{w}|} = \frac{1}{2} \sec \frac{\beta}{2} (\vec{v} + \vec{w})$$

$$\vec{z} = \frac{\vec{w} + \vec{u}}{|\vec{w} + \vec{u}|} = \frac{1}{2} \sec \frac{\gamma}{2} (\vec{w} + \vec{u})$$

$$\text{since } [\vec{x} \times \vec{y} \quad \vec{y} \times \vec{z} \quad \vec{z} \times \vec{x}] = [\vec{x} \quad \vec{y} \quad \vec{z}]^2$$

$$= \frac{1}{64} \sec^2 \frac{\alpha}{2} \sec^2 \frac{\beta}{2} \sec^2 \frac{\gamma}{2} [\vec{u} + \vec{v} \quad \vec{v} + \vec{w} \quad \vec{w} + \vec{u}]^2 \dots (1)$$

$$[\vec{u} + \vec{v} \quad \vec{v} + \vec{w} \quad \vec{w} + \vec{u}]^2 = 2[\vec{u} \quad \vec{v} \quad \vec{w}]^2$$

$$[\vec{x} \times \vec{y} \quad \vec{y} \times \vec{z} \quad \vec{z} \times \vec{x}] =$$

$$= \frac{1}{64} \sec^2 \frac{\alpha}{2} \sec^2 \frac{\beta}{2} \sec^2 \frac{\gamma}{2} \times 4[\vec{u} \quad \vec{v} \quad \vec{w}]^2$$

$$= \frac{1}{16} \sec^2 \frac{\alpha}{2} \sec^2 \frac{\beta}{2} \sec^2 \frac{\gamma}{2} [\vec{u} \quad \vec{v} \quad \vec{w}]^2$$

$$\text{Sol.10 (a)} \quad \text{Let } \vec{a} = (5, 2, 6)$$

$$\vec{b} = (2, 1, 1)$$

$$\vec{c} = (1, -1, 1)$$

Required unit vector will be along $= \vec{a} \times (\vec{b} \times \vec{c})$

$$\vec{a} \times (\vec{b} \times \vec{c}) = 27\hat{j} - 9\hat{k} \Rightarrow \text{unit vector} = \frac{3\hat{j} - \hat{k}}{\sqrt{10}}$$

$$\text{(b)} \quad \vec{a} = (1, 1, 1) \quad \vec{a} \cdot \vec{b} = 1$$

$$\vec{a} \times \vec{b} = \hat{j} - \hat{k}$$

$$\vec{a} \times (\vec{a} \times \vec{b}) = (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}$$

$$(\hat{i} + \hat{j} + \hat{k}) \times (\hat{j} - \hat{k}) = \vec{a} - 3\vec{b}$$

$$-2\hat{i} + \hat{j} + \hat{k} = \hat{i} + \hat{j} + \hat{k} - 3\vec{b}$$

$$\vec{b} = \hat{i}$$

$$\text{Sol.11} \quad \vec{a} \times \vec{b} = \vec{c} \times \vec{d} \quad \text{and} \quad \vec{a} \times \vec{c} = \vec{b} \times \vec{d}$$

$$(\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) \quad \text{and} \quad (\vec{c} \times \vec{d}) - (\vec{b} \times \vec{d})$$

$$\vec{a} \times (\vec{b} - \vec{c}) = (\vec{c} - \vec{b}) \times \vec{d}$$

$$\vec{a} \times (\vec{b} - \vec{c}) - (\vec{c} - \vec{b}) \times \vec{d} = 0$$

$$\vec{a} \times (\vec{b} - \vec{c}) - \vec{d} \times (\vec{b} - \vec{c}) = 0$$

$$(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = 0$$

$$\vec{a} - \vec{d} \text{ is } || \text{ to } (\vec{b} - \vec{c})$$

$$(\vec{a} - \vec{d}) \cdot (\vec{b} - \vec{c}) \neq 0$$

$$\vec{a} \cdot \vec{b} + \vec{d} \cdot \vec{c} \neq \vec{d} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\text{Sol.12} \quad \vec{a} \text{ is external angle}$$

$$\text{Bisector of } \hat{v} \text{ and } \hat{w} \quad \therefore \hat{w} - \hat{v} = \lambda \hat{a}$$

$$\text{Take modulus } |\hat{w} - \hat{v}|^2 = \lambda^2 |\hat{a}|^2$$

$$1 + 1 - 2 \hat{w} \cdot \hat{v} = \lambda^2$$

$$2 - 2 \cos 2\theta = \lambda^2$$

$$\lambda = 2 \sin \theta$$

$$\text{Hence, } \hat{w} - \hat{v} = 2 \sin \theta \hat{a}$$

$$= 2 \cos (90 - \theta) \hat{a} = - (2 \hat{a} \cdot \hat{v}) \hat{a}$$

$$\hat{w} = \hat{v} - 2 (\hat{a} \cdot \hat{v}) \hat{a}$$

$$\text{Sol.13 (a)} \quad \text{Let vector } \vec{r} \text{ be coplanar to } \vec{a} \text{ and } \vec{b}$$

$$\vec{r} = \vec{a} + t\vec{b}$$

$$\vec{r} = (1, 2, 1) + t(1, -1, 1)$$

$$\vec{r} = (1, 2, 1) + t(1, -1, 1)$$

$$\vec{r} = \hat{i}(1+t) + \hat{j}(2-t) + \hat{k}(1+t)$$

$$\text{Projection of } \vec{r} \text{ on } \vec{c} = \frac{1}{\sqrt{3}} \quad (\text{Given})$$

$$t = 3, \quad \vec{r} = (4, -1, 4)$$

$$\text{(b)} \quad \text{Vector } \overrightarrow{AB} \text{ is parallel to}$$

$$[(2\hat{i} + 3\hat{k}) \times (4\hat{j} - 3\hat{k})] \times [(\hat{j} - \hat{k}) \times (3\hat{i} + 3\hat{j})]$$

$$= 54 (\hat{j} - \hat{k})$$

Let θ is the angle between the vector, then

$$\cos \theta = \pm \left(\frac{54 + 108}{(3)(54)\sqrt{2}} \right) \pm \frac{1}{\sqrt{2}}$$

$$\text{Hence } \theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

Sol.14 (a) Given vectors are coplanar

$$\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0 \Rightarrow \lambda = \pm \sqrt{2}$$

(b) $\vec{a}, \vec{b}, \vec{c}$ are unit vectors.

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ represent an equilateral triangle

$$\therefore \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$$

(c) Statement-1

$$\vec{RS} + \vec{ST} = \vec{RT}$$

$\therefore \vec{PQ} \times \vec{RT} \neq 0$ as they are not collinear

Statement-1 is true.

Statement-2

$\vec{PQ} \times \vec{RS} = 0$ is false as \vec{PQ} and \vec{RS} are not parallel.

also $\vec{PQ} \times \vec{ST} = 0$ as they are parallel.

$\therefore \vec{PQ} \times \vec{ST} \neq 0$ is false.

Statement-2 is false.

Sol.15 (a) Volume = $[\hat{a} \hat{b} \hat{c}]$

$$[\hat{a} \hat{b} \hat{c}]^2 = \begin{vmatrix} \hat{a} \cdot \hat{a} & \hat{a} \cdot \hat{b} & \hat{a} \cdot \hat{c} \\ \hat{b} \cdot \hat{a} & \hat{b} \cdot \hat{b} & \hat{b} \cdot \hat{c} \\ \hat{c} \cdot \hat{a} & \hat{c} \cdot \hat{b} & \hat{c} \cdot \hat{c} \end{vmatrix} = \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix} = \frac{1}{2}$$

$$[\hat{a} \hat{b} \hat{c}] = \frac{1}{\sqrt{2}} \Rightarrow \text{Volume} = \frac{1}{\sqrt{2}}$$

(b) Given $\vec{OP} = \hat{a} \cos t + \hat{b} \sin t$

$$\vec{OP} = \sqrt{(\hat{a} \cdot \hat{a}) \cos^2 t + (\hat{b} \cdot \hat{b}) \sin^2 t + 2\hat{a} \cdot \hat{b} \sin t \cos t}$$

$$\vec{OP} = \sqrt{1 + \hat{a} \cdot \hat{b} \sin 2t}$$

$$|\vec{OP}|_{\max} = M = \sqrt{1 + \hat{a} \cdot \hat{b}} \text{ at } \sin 2t = 1 \Rightarrow t = \frac{\pi}{4}$$

$$\text{At } t = \frac{\pi}{4}; \quad \vec{OP} = \frac{1}{\sqrt{2}} (\hat{a} + \hat{b})$$

$$\text{unit vector along } \vec{OP} = \vec{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$$

Sol.16 (a) $[(\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}] \cdot \vec{d} = 1$

$$\left(\frac{\vec{b} \cdot \vec{d}}{2} \right) - (\vec{b} \cdot \vec{c})(\vec{a} \cdot \vec{d}) = 1$$

$$\left(\frac{\vec{b} \cdot \vec{d}}{2} \right) - 1 = 1 \Rightarrow \vec{b} \cdot \vec{d} = 4$$

$\Rightarrow \vec{b}$ & \vec{d} are non-parallel

(b)

(A) $2\sin^2\theta + 4\sin^2\theta \cos^2\theta = 2$

$$\sin^2\theta + 2\sin^2\theta (1 - \sin^2\theta) = 1$$

$$3\sin^2\theta - 2\sin^4\theta - 1 = 0$$

$$\Rightarrow \sin\theta = \pm \frac{1}{\sqrt{2}}, \pm 1$$

$$\theta = \frac{\pi}{4}, \frac{\pi}{2}$$

(B) Let $y = \frac{3x}{\pi}$

$$\Rightarrow \frac{1}{2} \leq y \leq 3 \quad \forall x \in \left[\frac{\pi}{6}, \pi \right]$$

$$\text{Now } f(y) = [2y] \cos [y]$$

$$\text{Critical points are } y = \frac{1}{2}, y = 1, y = \frac{3}{2}, y = 3$$

$$\text{Points of discontinuity } \left\{ \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \pi \right\}$$

(C) Volume = $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix}$

(D) $\vec{a} + \vec{b} = -\sqrt{3} \vec{c} \Rightarrow |\vec{a} + \vec{b}|^2 = \sqrt{3}$

$$|\vec{a} + \vec{b}|^2 = 3 \Rightarrow 1 + 1 + 2 \cos \theta = 3$$

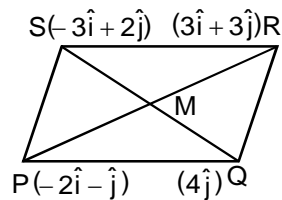
$$\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

Sol.17 Evaluating mid point of PR & QS which given M

$$= \left[\frac{\hat{i}}{2} + \hat{j} \right], \text{ same for both}$$

$$SR = PQ = \sqrt{37}$$

$$PS = QR = \sqrt{10}$$



angle b/w PS & SR $\neq 90^\circ$

angle b/w PR & SQ $\neq 90^\circ$

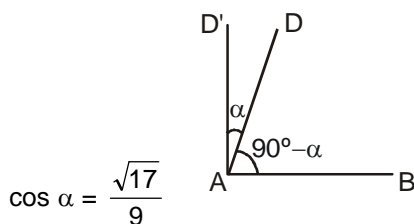
\Rightarrow only parallelogram.

Sol.18 5

$$\begin{aligned} & 2\vec{a} \cdot [(\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b})] + \vec{b} \cdot [(\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b})] \\ &= 2[\vec{a} \times (\vec{a} \times \vec{b}) \cdot (\vec{a} - 2\vec{b})] + [\vec{b} \times (\vec{a} \times \vec{b}) \cdot (\vec{a} - 2\vec{b})] \\ &= -2\vec{b} \cdot (\vec{a} - 2\vec{b}) + (\vec{a}) \cdot (\vec{a} - 2\vec{b}) = 4|\vec{b}|^2 + |\vec{a}|^2 \\ &(\vec{a} \cdot \vec{b} = 0) \end{aligned}$$

Sol.19 B

$$\sin(90^\circ - \alpha) = \frac{|\vec{AB} \times \vec{AD}|}{|\vec{AB}| |\vec{AD}|}$$



$$\cos \alpha = \frac{\sqrt{17}}{9}$$

Sol.20 C

Let the vector is \vec{v}

$$\vec{v} = x\vec{a} + y\vec{b}$$

$$\vec{v} = (x + y)\hat{i} + (x - y)\hat{j} + (x + y)\hat{k} \quad \dots(1)$$

$$\text{proj}^n \text{ of } \vec{c} = \frac{\vec{v} \cdot \vec{c}}{|\vec{c}|} = \frac{(x + y) - (x - y) - (x + y)}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \Rightarrow x - y = -1 \quad \dots(2)$$

Sol.21 A,D

Required vector

$$= [(\hat{i} + \hat{j} + 2\hat{k}) \times (\hat{i} + 2\hat{j} + \hat{k})] \times (\hat{i} + \hat{j} + \hat{k})$$

$$= k(4\hat{j} + 4\hat{k}) = \lambda(\hat{i} - \hat{k}) \text{ so } A \& D$$

Sol.22

$$(\vec{r} - \vec{c}) \times \vec{b} = 0$$

$$\Rightarrow \vec{r} = \vec{c} + \lambda \vec{b} = (1 - \lambda, 2 + \lambda, 3)$$

$$\vec{r} \cdot \vec{a} = 0 \Rightarrow \lambda = 4 \text{ so } \vec{r} \cdot \vec{b} = 9$$

Sol.23 C

$$|\vec{a} + \vec{b}| = \sqrt{29}$$

$$\vec{a} \times (2, 3, 4) - (2, 3, 4) \times \vec{b} = 0$$

$$\vec{a} \times (2, 3, 4) + \vec{b} \times (2, 3, 4) = 0$$

$$(\vec{a} + \vec{b}) \times (2, 3, 4) = 0$$

$$\vec{a} + \vec{b} = \lambda(2, 3, 4)$$

$$|\vec{a} + \vec{b}| = \sqrt{29}$$

$$|\vec{a} + \vec{b}|^2 = 29$$

$$\lambda^2(4 + 9 + 16) = 29$$

$$\lambda^2 = 1 \Rightarrow \lambda = \pm 1$$

$$\begin{aligned} (\vec{a} + \vec{b}) \cdot (-7, 2, 3) &= \lambda(2, 3, 4) \cdot (-7, 2, 3) \\ &= \lambda[-14 + 6 + 12] \\ &= \lambda[4] \\ &= 4, -4 \end{aligned}$$

Sol.24 Given is maximum value of expression & hence angle b/w any two is 120°

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = \frac{-1}{2}$$

$$\text{so } |2\vec{a} + 5\vec{b} + 5\vec{c}| \text{ is } 3$$